[2] $\ln(1-x+x^2) = \ln\left(\dfrac{1+x^3}{1+x}\right) = \boxed{\ln(1+x^3) - \ln(1+x)}$ 1

$$\approx \boxed{\left(x^3 - \frac{(x^3)^2}{2}\right) - \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6}\right)} \quad \frac{1}{2}$$

$$\approx \boxed{-x + \frac{x^2}{2} + \frac{2x^3}{3} + \frac{x^4}{4} - \frac{x^5}{5} - \frac{x^6}{3}} \quad \frac{1}{2}$$

[3] $2(4-x^3)^{-\frac{3}{2}} = 2\cdot 4^{-\frac{3}{2}}\left(1-\frac{x^3}{4}\right)^{-\frac{3}{2}}$

$$= \boxed{\frac{1}{4}\sum_{n=0}^{\infty}\binom{-\frac{3}{2}}{n}\left(-\frac{x^3}{4}\right)^n} \quad 1$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n(2n+1)!}{2^{2n}\cdot(n!)^2}\cdot\frac{(-1)^n x^{3n}}{4^{n+1}}$$

$$= \boxed{\sum_{n=0}^{\infty}\frac{(2n+1)!}{2^{4n+2}(n!)^2}x^{3n}} \quad 1$$

$(1+x)^k$ HAS RADIUS 1

IE. CONVERGES IF $-1<x<1$

$\left(1-\frac{x^3}{4}\right)^{-\frac{3}{2}}$ CONVERGES IF $\boxed{-1<-\frac{x^3}{4}<1}$ 1

$$4>x^3>-4$$

$$-\sqrt[3]{4}<x<\sqrt[3]{4}$$

RADIUS = $\boxed{\sqrt[3]{4}}$ $\frac{1}{2}$

$$4^{-\frac{3}{2}} = \frac{1}{(\sqrt{4})^3} = \frac{1}{8} \quad \frac{1}{2}$$

$$\binom{-\frac{3}{2}}{n} = \boxed{\frac{(-\frac{3}{2})(-\frac{5}{2})(-\frac{7}{2})\cdots(-\frac{3}{2}-n+1)}{n!}}$$

$$= \frac{(-1)^n(3\cdot5\cdot7\cdots(2n+1))}{2^n\cdot n!}$$

$$= \frac{\boxed{(-1)^n\frac{(2n+1)!}{2^n\cdot n!}}}{2^n\cdot n!} \quad 1$$

$$= \boxed{\frac{(-1)^n(2n+1)!}{2^{2n}\cdot(n!)^2}} \quad 1$$

NOTE: $3\cdot5\cdot7\cdots(2n+1)$

$$= 3\cdot5\cdot7\cdots(2n-1)(2n+1)$$

$$= \frac{(2n)!}{2^n n!}\cdot(2n+1)$$

$$= \frac{(2n+1)!}{2^n\cdot n!}$$

[4] [a]

n	$f^{(n)}(x)$	
0	x^{-2}	$= (-1)^0 1! x^{-2}$
1	$-2x^{-3}$	$= (-1)^1 2! x^{-3}$
2	$(-2)(-3)x^{-4}$	$= (-1)^2 3! x^{-4}$
3	$(-2)(-3)(-4)x^{-5}$	$= (-1)^3 4! x^{-5}$
n	$(-2)(-3)(-4)\cdots(-(n+1))x^{-(n+2)}$	$= \boxed{(-1)^n (n+1)! x^{-n-2}}$ $\frac{1}{2}$

$\frac{1}{2}$

$$f^{(n)}(3) = (-1)^n (n+1)!\, 3^{-n-2}$$

$$T(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (n+1)!\, 3^{-n-2}}{n!}(x-3)^n = \boxed{\sum_{n=0}^{\infty} (-1)^n (n+1) 3^{-n-2} (x-3)^n}$$ 1

[b] $$\lim_{n\to\infty} \left| \frac{(-1)^{n+1}(n+2)3^{-n-3}(x-3)^{n+1}}{(-1)^n (n+1) 3^{-n-2}(x-3)^n} \right| = \lim_{n\to\infty} \left|\frac{n+2}{n+1}\right| 3^{-1}(x-3)$$

$$= \boxed{\frac{1}{3}|x-3| \lim_{n\to\infty} \left| \frac{1+\frac{2}{n}}{1+\frac{1}{n}} \right|} \quad \frac{1}{2}$$

$$= \frac{1}{3}|x-3| < 1 \longrightarrow \boxed{-1 < \frac{1}{3}(x-3) < 1} \quad \frac{1}{2}$$

$$-3 < x-3 < 3$$

CONVERGES IF $\boxed{0 < x < 6}$ $\frac{1}{2}$

$x = 0$

$$\sum_{n=0}^{\infty} (-1)^n (n+1) 3^{-n-2} (-3)^n$$

$$= \sum_{n=0}^{\infty} (n+1) 3^{-n-2} 3^n$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{n+1}{9}} \quad \frac{1}{2}$$

$$\boxed{\lim_{n\to\infty} \frac{n+1}{9} = \infty \neq 0} \quad \frac{1}{2}$$

SERIES $\boxed{\text{DIVERGES}}$ FOR $x=0$ $\boxed{\text{(DIV TEST)}}$ $\frac{1}{2}$

$x = 6$

$$\sum_{n=0}^{\infty} (-1)^n (n+1) 3^{-n-2} 3^n$$

$$= \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{n+1}{9}} \quad \frac{1}{2}$$

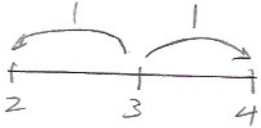
$$\boxed{\lim_{n\to\infty} \left| (-1)^n \frac{n+1}{9} \right| = \lim_{n\to\infty} \frac{n+1}{9} = \infty \neq 0}$$

$$\text{SO } \boxed{\lim_{n\to\infty} (-1)^n \frac{n+1}{9} \neq 0} \quad \frac{1}{2}$$

SERIES $\boxed{\text{DIVERGES}}$ FOR $x=6$ $\boxed{\text{(DIV TEST)}}$ $\frac{1}{2}$

INTERVAL OF CONVERGENCE $= \boxed{(0, 6)}$ $\frac{1}{2}$

[c] $f^{(n+1)}(x) = (-1)^{n+1}(n+2)!\, x^{-n-3}$



$$|f^{(n+1)}(x)| = |(-1)^{n+1}(n+2)!\, x^{-n-3}|$$

$$= (n+2)!\, |x^{-n-3}|$$

$$\le \boxed{(n+2)!\, 2^{-n-3} = M} \quad 1$$

FOR $|x-3| \le 1$ IE. $2 \le x \le 4$

x^{-n-3} IS LARGEST AT $x=2$

SO $0 < x^{-n-3} \le 2^{-n-3}$

IE. $|x^{-n-3}| \le 2^{-n-3}$

$$|R_n(x)| \le \frac{(n+2)!\, 2^{-n-3}}{(n+1)!} |x-3|^{n+1} = \boxed{(n+2) 2^{-n-3} |x-3|^{n+1}} \quad \frac{1}{2}$$

$$|R_n(4)| \le 10^{-10} \text{ IF } \boxed{(n+2) 2^{-n-3} \le 10^{-10}} \quad 1$$

$$10^{10}(n+2) \le 2^{n+3}$$

$$n \ge 36$$

$$\boxed{\text{MINIMUM } n = 36} \quad \frac{1}{2}$$